Linear - Quiz 3

9:03

9:13

10 → give 20 minutes

Name: Key

1. (4 points) Let A be a 3×4 matrix with 2 pivot positions. Answer the following and explain (briefly) your reasoning:

   (a) Does $A\mathbf{x} = 0$ have a nontrivial solution? $\boxed{\text{Yes}}$

$$\begin{bmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

If there are 2 pivot positions then there will be two free variables. Free variables ⇒ nontrivial solutions. (i.e. ∞ # of solutions)

   (b) Does $A\mathbf{x} = \mathbf{b}$ have at least one solution for every possible $\mathbf{b}$?

$\boxed{\text{No}}$. In reduced form, the augmented matrix $[A \vdots b]$ will be of the form $\left[\begin{array}{cccc|c} 1 & 0 & * & * & * \\ 0 & 1 & * & * & * \\ 0 & 0 & 0 & 0 & * \end{array}\right]$.

If b is such that this entry is $\neq 0$ then the system is inconsistent and there will be no solution.

2. (4 points) Find the value(s) of h for which the vectors are linearly dependent. Justify (briefly) your answer.

$$\begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 \\ 5 & -9 & h \\ -3 & 6 & -9 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & h-15 \\ 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Ans: } \underline{\text{all}} \text{ values of } h}$

So the 3rd column has no pivot and thus there is a free variable ⇒ the vectors will be linearly dependent no matter what the value of h.

3. (2 points) Let T be defined by $T(x) = A\mathbf{x}$, where $A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}$. Find a vector $\mathbf{x}$ whose image under T is b=$\begin{bmatrix} 4 \\ 4 \end{bmatrix}$.

$$\left[\begin{array}{ccc|c} 1 & -5 & -7 & 4 \\ -3 & 7 & 5 & 4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -5 & -7 & 4 \\ 0 & -8 & -16 & 16 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 3 & -6 \\ 0 & 1 & 2 & -2 \end{array}\right]$$

$$\to x_1 + 3x_3 = -6$$
$$x_2 + 2x_3 = -2$$

Let $x_3 = 0$, get $x_1 = -6$, $x_2 = -2$

⇒ $\boxed{x = \begin{bmatrix} -6 \\ -2 \\ 0 \end{bmatrix} \text{ is an answer (there are an infinite \# of answers)}}$